

# HEAT AND MASS TRANSFER FOR REACTION OF INJECTED FLUID WITH EXTERNAL OXYGEN FLOW

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**Аннотация**—Приводится теоретическое и экспериментальное исследование тепло-и массообмена в ламинарном пограничном слое при подаче через пористую стенку жидкого инжектанта (гексана или этилового спирта), реагирующего после испарения с кислородом внешнего потока. Система дифференциальных уравнений переноса в пограничном слое решается для случая бесконечно большой скорости гомогенной реакции с учетом многокомпонентной диффузии при переменных и не равных друг другу числах  $Pr$  и  $Sc_s$ . Экспериментальным путем измерялись поля температур и концентраций в пограничном слое четырехсекционной пористой пластины, затем определялась интенсивность локального и среднего по длине тела тепло-и массообмена.

## NOMENCLATURE

$x, y,$	coordinates along plate and normal to it;	$h,$	$= \sum_{i=1}^N n_i h_i;$
$u, v,$	velocities of flow along $x$ and $y$ , respectively;	$H,$	total enthalpy ( $H = h + u^2/2$ );
$R,$	universal gas constant;	$Pr, Sc_i, Le_i,$	Prandtl, Schmidt and Lewis numbers where $Pr = \mu \bar{c}_p / \pi,$ $Sc_i = \mu / \rho D_i$ (diffusional Prandtl number $Pr_{mi} \equiv Sc_i$ ), $Le_i = Pr / Sc_i$ ;
$T,$	temperature;	$St, St_m, Re,$	local Stanton numbers determined from (28) and the Reynolds number ( $Re = \rho_\infty u_\infty x / \mu_\infty$ );
$c_{pi}, n_i, D_i,$	specific heat at constant pressure, concentration and diffusion coefficient of the $i$ th gas component, respectively;	$\bar{St}, \bar{St}_m, \bar{Re},$	mean numbers referred to the whole length of plate $L$ ;
$D_{ik}$	binary diffusion coefficient;	$K,$	$= (\pi_w / \pi_*) (T_\infty / T_*)$ ;
$\rho, \pi, c_p, \mu,$	density, coefficients of thermal conductivity, heat capacity and dynamic viscosity (of gas mixture), respectively;	$\eta,$	$= u / u_\infty$ ;
$m,$	molecular weight;	$\tilde{\rho},$	$= \rho / \rho_\infty$ ;
$r,$	heat of evaporation;	$\tilde{\mu},$	$= \mu / \mu_\infty$ ;
$j_i,$	diffusional mass flow ( $j_i = -\rho D_i n_{iy}$ );	$\beta_{ij}$	number of atoms of element $j$ in molecule of component $i$ .
$\bar{j}_i,$	normalized mass flow ( $\bar{j}_i = -Sc_i \partial n_i / \partial \eta$ );	<b>Subscripts</b>	
$h_i,$	enthalpy of the $i$ th component ( $h_i = \int_{T_0}^T c_{pi} dT + \Delta i$ );	$\infty,$	conditions outside the boundary layer;
$\Delta i,$	enthalpy of formation of the $i$ th component at $T = T_0$ ;	$w,$	conditions at the surface;
		$*$ ,	conditions within the reaction region;
		$i$ or $k,$	components;
		$j,$	elements of gas;

$x, y,$	derivatives with respect to $x$ or $y$ ;
'	derivatives with respect to $\eta$ ;
$cr,$	critical;
$pol,$	polar;
$dr,$	dry;
I,	value between reaction front and wall;
II,	value over reaction front in boundary layer;
$a, h, o, d, wat, c, n,$	$C_2H_5OH, C_6H_{14}, O_2, CO_2, H_2O, CO$ and $N_2$ , respectively;
$S,$	injected fluid ( $S \equiv a$ or $S \equiv h$ ).

AN INCREASING interest in the study of the problem of a reacting boundary layer can be explained by its great practical value. The study of heat and mass-transfer process with a phase and of chemical conversions is connected with the solution of a number of problems in modern engineering, in particular, with the problem of porous cooling, design and calculation of various combustion chambers, furnaces, high-temperature heat exchangers, reaction engine nozzles, etc.

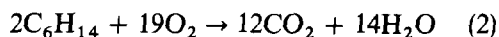
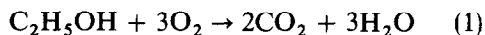
There are available many theoretical works [1] on the above problem. The experimental investigation mainly considers ablation or sublimation of various materials [1], however the experimental data on heat and mass transfer for the reaction of an oxidising agent in an external flow with liquid or gas injected into the surface of a body through a porous wall are published only in a few works [2]. Thus Kulgein [2] determined only heat and mass-transfer parameters along the body length (with a longitudinal flow around a cylinder of 38 mm dia. and 914 mm length).

The present work deals with experimental and theoretical study of heat- and mass-transfer process in a laminar boundary layer on a multi-sectional porous plate for reacting (burning) vapours of liquid (ethyl alcohol  $C_2H_5OH$  or hexane  $C_6H_{14}$ ) filtered through the wall into an external oxygen flow.

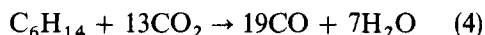
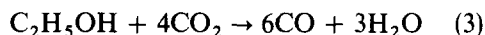
The analytic study of the problem considered

is carried out for the case of an infinitely fast rate of a homogeneous reaction proceeding both in a clearly marked region (reaction front)  $y = y_*$  ( $x$ ), which is the break surface in the boundary layer, and near the surface of the body. It may be considered that on the surface of the body an active component (ethyl alcohol or hexane vapours) reacts with carbon dioxide under conditions of the absence of oxygen [3].

According to the adopted mechanism of the process, one of the reactions



proceeds within the region  $y = y_*(x)$ , and the other



on the surface of the body respectively.

Reactions (1)–(4) occur in the presence of nitrogen  $N_2$  which is considered chemically inert. It is assumed that the products of reactions (1) and (2), i.e.  $CO_2$  and  $H_2O$  diffuse both to the surface of the body and to the external edge of the boundary layer where their concentrations tend to zero. On the surface of the body reactions (3) and (4) yield  $CO$  and  $H_2O$ , with  $CO$  being oxidized to  $CO_2$  in the region  $y = y_*(x)$ . Thus, when both types of reactions take place, gaseous 5 components ( $N_2, H_2O, CO_2, CO$  and  $C_6H_{14}$  or  $C_2H_5OH$ ) will be present in the 1st region ( $0 \leq y \leq y_*$ ), and 4 components ( $N_2, H_2O, CO_2$  and  $O_2$ )† in the 2nd. It is also assumed that in the 1st region oxygen in molecular form is absent and in the 2nd components  $a$  and  $h$  are absent. Within the reaction region,  $y = y_*(x)$ , the concentration of oxygen  $n_o$  and of vapours of injected component (ethyl alcohol  $n_a$  or hexane  $n_h$ ) is zero; concentration of carbon monoxide  $n_c$  at the break surface is also negligibly low. In addition the assumption is made

† Components  $C_2H_5OH, C_6H_{14}, O_2, CO_2, H_2O, CO$  and  $N_2$  are subsequently denoted by subscripts  $a, h, o, d, wat, c$  and  $n$ , respectively.

that

$$H_x = n_x = 0. \tag{5}$$

Since the injection considered further occurs in accordance with the law  $(\rho v)_w = \text{const}$ , assumption (5) does not present any limitations to the statement of the problem.

It should be noted that the adopted analytic representation of the process based on the assumption of diffusional burning is only approximately valid especially for the low temperature region near the surface of the body which is not catalytic. Reactions (3) and (4) proceed in the wall region at finite rates the incorporation of which presents great mathematical difficulties, the more so since no information about the kinetic rate constants of these reactions is available. Hence, it is subsequently assumed that reactions of the form (3) and (4) proceed on the surface of the body with partial conversion of gas CO<sub>2</sub> to CO. The relationships between concentrations of CO<sub>2</sub> and CO on the wall may be found from experiment.

As reactions (3) and (4) occur in the region of maximum concentrations of injected fluid, i.e. within the wall region, the assumption about reactions (3) and (4) on the wall may be regarded as acceptable.

The equations of conservation of momentum, continuity of the *i*th component and energy for a laminar boundary layer of a reacting gas mixture consisting of *N* components (*N*<sub>I</sub> = 5, *N*<sub>II</sub> = 4) with incorporation of (5) and disregarding thermal diffusion may be presented in terms of the Crocco variables [4]  $x = x$ ,  $\eta = u(x, y)/u_\infty$  in the form (prime refers to derivatives with respect to  $\eta$ )

$$\omega\omega'' + 2\tilde{\mu}\rho\eta = 0 \tag{6}$$

$$\left(\frac{n_i}{Sc_i}\right)' + (1 - Sc_i)\frac{\omega'}{\omega}\left(\frac{n_i}{Sc_i}\right) = 0, \tag{7}$$

$$i = 1, 2, \dots, N$$

$$\left(\frac{H'}{Pr}\right)' + (1 - Pr)\frac{\omega'}{\omega}\left(\frac{H}{Pr}\right) = \zeta(\eta) \tag{8}$$

where

$$\omega = 2\tau\sqrt{(x/\rho_\infty\mu_\infty u_\infty^3)}, \quad \tau = \mu u_y$$

$$\zeta(\eta) = -\frac{1}{Pr}\frac{\omega'}{\omega}\varphi_z(\eta) - \left[\frac{1}{Pr}\varphi_z(\eta)\right]', \quad z = I, II$$

$$\varphi_I(\eta) = A - h_S(n_S + Prj_S)$$

$$\varphi_{II} = u_\infty^2\eta(Pr - 1) - h_O(n'_O + Prj'_O) - h_d(n'_d + Prj'_d) - h_{wat}(n'_{wat} + Prj'_{wat}) - h_n(n'_n + Prj'_n),$$

$$A = u_\infty^2\eta(Pr - 1) - h_c(n'_c + Prj'_c) - h_d(n'_d + Prj'_d) - h_{wat}(n'_{wat} + Prj'_{wat}) - h_n(n'_n + Prj'_n),$$

$$\tilde{j}_i = -n'_i/Sc_i.$$

Equation (7) does not include the term  $w_i$  characterizing the power of sources and sinks of the substance due to chemical reactions since a chemical reaction is considered proceeding at a finite rate in clearly marked regions: on the surface of the body and in the boundary layer at  $\eta = \eta_*(x)$ . The rate of component formation  $w_i$  will be taken into account by the appropriate boundary conditions.

Equations (6)–(8) should be completed by the state equation

$$P = R\rho T \sum_{i=1}^N n_i/m_i \tag{9}$$

and the relationship

$$\sum_{i=1}^N n_i = 1. \tag{10}$$

The boundary conditions for (6)–(8) are written as:

at  $\eta = 0$

$$\omega' = 2u_\infty(\rho v)_w\sqrt{(x/\rho_\infty\mu_\infty u_\infty^3)}; \quad H = H_w \tag{11}$$

$$n_i = n_{iw}, \quad i = S, d, wat, c$$

at  $\eta = 1$

$$\omega = 0; H = H_\infty; n_{i\infty} = 0; i = d, wat; \tag{12}$$

$$n_0 = n_{0\infty}$$

at  $\eta = \eta_*$

$$H = H_*; n_i = 0, i = O, c, S, n_i = n_{i*}, \quad (13)$$

$$i = d, wat$$

$$(\pi T')_{II} - (\pi T')_I = Q_s j_s$$

where  $Q_s$  is the heat of reaction per unit mass of injected fluid  $S$  (ethyl alcohol or hexane). It should be noted that for reactions (1)–(3) in relationships (6)–(13)  $S \equiv a$ , for (2), (4)  $S \equiv h$ .

Problem (6)–(13) should be supplied with the following  $(N - 1)$  expressions for regions I and II which establish the relationships between normalized flows  $j_i$ , binary diffusion coefficients, molecular weights  $m_i$  and concentrations  $n_i$  [7]

$$\frac{\mu}{\rho} \sum_{k=1}^N \frac{n^k}{m_k D_{ik}} \left( \frac{j_i}{n_i} - \frac{j_k}{n_k} \right) = \sum_{k=1}^N \frac{n_k}{m_k} \left( \frac{n'_k}{n_k} - \frac{n'_i}{n_i} \right),$$

$$\sum_{i=1}^N j_i = 0 \quad k \neq i, i = 1, 2, \dots, N \quad (14)$$

here  $m_c = m_w$ ,  $D_{Sc} = D_{Sw}$ ,  $DD_{dc} = D_{dw}$ ,  $D_{watc} = D_{waw}$ .

Physical properties of individual components were determined according to [5] from formulae of the form  $\mu_i = \mu_i(m_i, T, \sigma_i)$ ,  $\pi = \pi_i = \pi_i(\mu_i, m_i, c_{vi})$ ,  $c_{pi} = c_{pi}(T)$ ,  $D_{ik} = D_{ik}(T, m_i, m_k, \sigma_{ik})$ , where  $\sigma_i$  is the diameter of the  $i$ th component molecule.

$$\sigma_{ik} = (\sigma_i + \sigma_k)/2.$$

For gas mixtures, coefficients  $\mu$ ,  $\pi$ ,  $\bar{c}_p$  were determined from relationships [6]

$$\mu = \frac{\sum_{i=1}^N n_i \mu_i (m_i T_{crit})^{0.5}}{\sum_{i=1}^N n_i (m_i T_{crit})^{0.5}},$$

$$\lambda = \left( 1 + \frac{n_{pol.i} - n_{pol.i}^2}{3.5} \right) \sum_{i=1}^N n_i \pi_i$$

$$\bar{c}_p = \sum_{i=1}^N n_i c_{pi}, i = 1, 2, \dots, N. \quad (15)$$

As follows from (6)–(15), a closed set is formed of differential equations, boundary and additional conditions which should be solved for the regions I and II. Within the reaction zone ( $\eta = \eta_*$ ) the values  $\xi \equiv u, v, \mu, \rho, T, n_i, \omega$  and  $\omega'$  must be continuous functions, but the values  $H', n'_i$  and  $j_i$ , discontinuous [7]. By integrating equations (7) and (8) term by term from  $(\eta_* - \delta)$  to  $(\eta_* + \delta)$  and taking into account that

$$\lim_{\delta \rightarrow 0} \xi(\eta_* + \delta) - \xi(\eta_* - \delta) = \xi|_I^{III} = 0$$

for the region  $\eta = \eta_*$ , the following equations are obtained, respectively

$$\sum_{i=1}^N \frac{\beta_{ij} m_j}{m_i} j_i \Big|_I^{III} = 0, j = 1, 2, \dots, L \quad (16)$$

$$H' \Big|_I^{III} = \sum_{i=1}^N h_i (n'_i + Pr j_i) \Big|_I^{III}. \quad (17)$$

Here  $\beta_{ij}$  is the number of atoms of element  $j$  ( $j = 1, 2, \dots, L$ ) in a molecule of component  $i$ , with  $j_o^I = j_o^{II} = j_o^{III} = 0$ . Besides, on the wall average mass rates  $J_i$  for  $H_2O$  and  $N_2$  are equal to zero, i.e.  $J_{iw} = (\rho v n_i)_w + j_{iw} = 0$ ,  $i = wat, n$ .

Condition (13) which can be obtained by way of simple transformations from (17) is further used for determination of the position of the reaction zone  $\eta_*$ . From equation (16) the values of unknown concentrations  $n_{i,w}$  on the surface of the body may be defined. The solution of the problem (6)–(13) may be obtained by numerical integration. However with the help of two quadratures (7) and (8) the following relationships for enthalpy and concentration distribution within the regions I and II, convenient for numerical calculation, may be obtained

$$\left. \begin{aligned} H_I &= H_w + [H_* - H_w - N(o, \eta_*)] \\ &\quad \times \sigma(o, \eta)/\sigma(o, \eta_*) + N(o, \eta), \\ H_{II} &= H_* + [H_\infty - H_* - N(\eta_*, 1)] \\ &\quad \times \sigma(\eta_*, \eta)/\sigma(\eta_*, 1) + N(\eta_*, \eta) \end{aligned} \right\} \quad (18)$$

$$\left. \begin{aligned} \frac{n_{ii} - n_{iw}}{n_{i*} - n_{iw}} &= \frac{\sigma_i(0, \eta)}{\sigma_i(0, \eta_*)}, i = d, wat, c, n, S \\ \frac{n_{iii} - n_{i*}}{n_{i\infty} - n_{i*}} &= \frac{\sigma_i(\eta_*, \eta)}{\sigma_i(\eta_*, 1)}, i = o, d, wat, n \end{aligned} \right\} (19)$$

where, according to the analytical calculation

$$q_w = \frac{\tau_w}{u_\infty} \left[ \frac{H_* - H_w - N(0, \eta_*)}{\sigma(0, \eta_*)} - \left( \frac{1}{Pr} \sum_{i=1}^N h_i n'_i \right)_w \right] \quad (22)$$

where  $n_{i*} = 0$  for components  $i \equiv a, h, c, o$

$$\left. \begin{aligned} N(\eta_1, \eta_2) &= \int_{\eta_1}^{\eta_2} \varphi(\eta_1, \eta_2) d\eta \\ \sigma(\eta_1, \eta_2) &= \int_{\eta_1}^{\eta_2} \psi(\eta_1, \eta_2) d\eta \\ \sigma_i(\eta_1, \eta_2) &= \int_{\eta_1}^{\eta_2} \psi_i(\tilde{\eta}_1, \tilde{\eta}_2) d\eta \\ \varphi(\eta_1, \eta_2) &= Pr \exp \left[ - \int_{\omega(\tilde{\eta}_1)}^{\omega(\eta_2)} (1 - Pr) \frac{d\omega}{\omega} \right] \\ &\times \int_{\tilde{\eta}_1}^{\eta_2} (\zeta_s(\eta) \exp \int_{\omega(\tilde{\eta}_1)}^{\omega(\eta_2)} (1 - Pr) \frac{d\omega}{\omega} d\eta \\ \psi(\eta_1, \eta_2) &= Pr \exp \left[ - \int_{\omega(\tilde{\eta}_1)}^{\omega(\eta_2)} (1 - Pr) \frac{d\omega}{\omega} \right], \\ \psi_i(\eta_1, \eta_2) &= Sc_i \exp \left[ - \int_{\omega(\tilde{\eta}_1)}^{\omega(\eta_2)} (1 - Sc_i) \frac{d\omega}{\omega} \right]. \end{aligned} \right\} (20)$$

where  $\tau_w = \omega(0)(\rho_\infty \mu_\infty u_\infty^3/x)^{1/2}$  and  $q_R, q_p, q_{sk}, q_d$  are wall heat fluxes due to radiation, phase changes and chemical reactions (2) and (4), respectively, per unit mass of the component  $d$ . The methods of calculation of  $q_R$  are presented in [3],  $q_p = z_s(\rho v)_{sw}$ ,  $q_d$  is determined for reactions (3) and (4) from the expressions

$$\begin{aligned} q_d &= (\frac{1}{4}m_d)(m_a h_a + 4m_d h_d - 6m_c h_c - 3m_{wat} h_{wat}), \\ q_d &= (\frac{1}{13}m_d)(m_h h_h + 13m_d h_d - 19m_c h_c - 7m_{wat} h_{wat}). \end{aligned}$$

From condition (13), using (18) and (19), the following algebraic expression for the calculation of the reaction front  $\eta_*$  position is found:

$$\begin{aligned} &[H_\infty - H_* - N(\eta_*, 1)]Pr/\sigma(\eta_*, 1) \\ &- \sum_{i(I)} h_i(n_{i\infty} - n_{i*}) Sc_i/\sigma_i(\eta_*, 1) \\ &- [H_* - H_w - N(0, \eta_*)]\psi(0, \eta_*) \\ &+ \sum_{i(II)} h_i(n_{i*} - n_{iw})\psi_i(0, \eta_*) - \varphi(0, \eta_*) \\ &- Q_s(\bar{c}_p/\pi) \psi_s(0, \eta_*)/\sigma_s(0, \eta_*) = 0 \quad (23) \end{aligned}$$

Solution of equation (6) for a number of cases is given, for example in [9].

The limits of integration of  $\eta_1$  and  $\eta_2$  in  $N(\eta_1, \eta_2), \sigma(\eta_1, \eta_2)$  and  $\sigma_i(\eta_1, \eta_2)$  have the values 0; 1;  $\eta_*$  and  $\eta$ . Analytical expressions (18) and (19) are the same for all the reactions (1)-(4).

Since

$$(\pi T_y)_w = (H' - \sum_{i=1}^N h_i n_i)_w \tau_w / \overline{Pr}_w$$

then, utilising (18), the local heat flux into the wall can be found

$$q_w = -\pi T_y = q_p + q_{sk} - q_{ch} - q_R \quad (21)$$

where for the 1st region  $i(I) = S, d, wat, c, n$ , and for the 2nd region,  $i(II) = o, d, wat, n$ .

Relationship (23), obtained on the basis of solution of differential equations for  $\omega, H$  and  $n_i$ , is the most general one for characterisation of the process under consideration. Therefore compared are (Fig. 1) only theoretical values of  $\eta_*$  obtained from (23) and experimental ones obtained from the position of the maximum on the temperature curves (Fig. 2). The agreement between curves 1 and 2 in Fig. 1 is satisfactory.

The experimental study of the process under consideration was carried out in a low-speed

wind tunnel described in [8]. The experimental body, the diagram of which is given in [8], was a porous ( $P = 30$  per cent) metal plate  $196 \times 60 \times 3$  mm divided into 4 sections. The first section was 30 mm long, the second 40 mm, the third and

fourth 60 mm each and the passive section 6 mm. The outer side of the porous plate was flush with the lower wall of the working section of the wind tunnel. The slit before the front section served for sucking in the boundary layer to avoid it being affected by the flow history. Nickel-coated copper tubes, 6 mm dia., through which cooling water was flowing, were mounted inside each section (under the porous plate). Branch tubes connected with glass measuring tubes were built into the end faces of the sections. Through these measuring tubes any vapour-air bubbles formed under the plate were removed

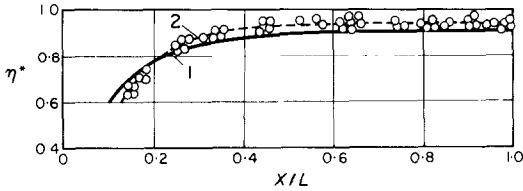


FIG. 1. Theoretical and experimental values of  $\eta_*$  as a function of  $x/L$ .

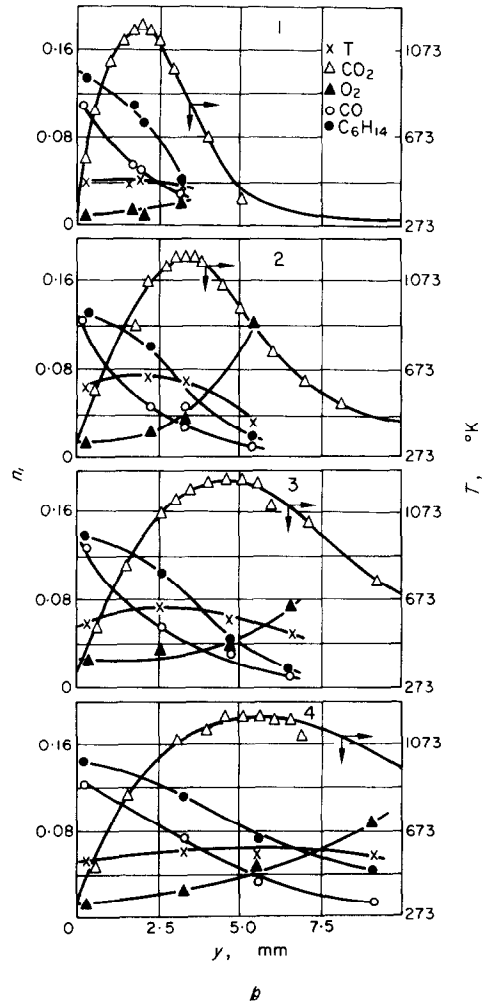
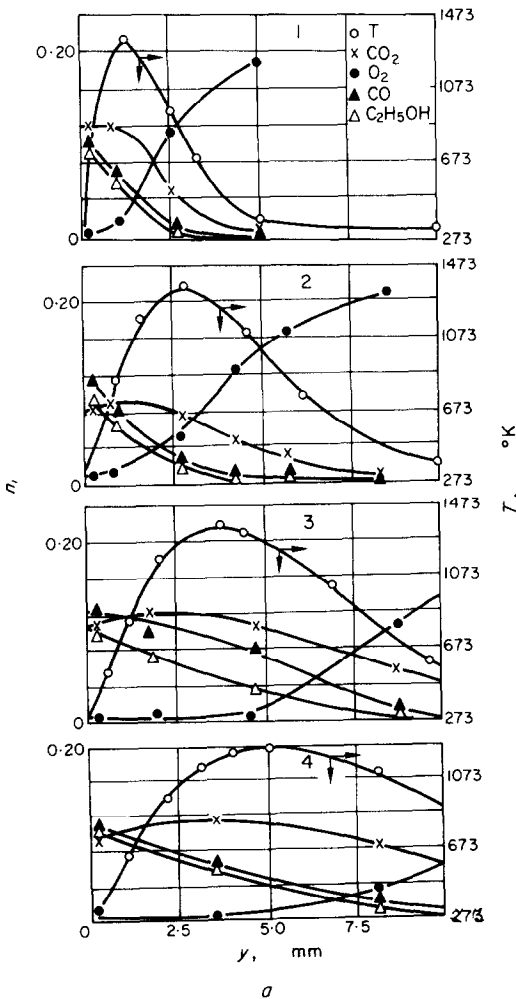


FIG. 2. Characteristic experimental fields of temperature and concentration for (a) ethyl alcohol and (b) hexane.

and the wires passed of the copper-constantan thermocouples which were then connected to a potentiometer. The required hydraulic head of the injected fluid and the position of the evaporation zone was maintained by suitable division of the tubes. The position of the evaporation zone was assumed to coincide with the surface of the porous plate. Eight thermocouples were built into each of the four sections (four of the thermocouples into the surface). The reacting liquid (ethyl alcohol or hexane) was supplied through connections and its consumption was determined by weighing.

In the experimental study the following local parameters were determined simultaneously across the height of the boundary layer: temperature, velocity and concentration. To do this the total pressure tube 0.43 mm dia. (it also served for withdrawing concentration samples), was connected to a chromel-alumel thermocouple (wires 0.15 mm dia.) and formed a measuring probe moved both in longitudinal and transverse directions with the help of a traversing gear—a micrometric screw with scale divisions 0.01 mm. The probe for measuring the static pressure was set near the experimental body. For measuring the flow velocity in the tube, a Pitot-Prandtl tube was used. The analysis of gas samples taken in the reacting boundary layer was done by a chromatograph, type XII-3.

Experiments were carried out for the cases of injections obeying the relation  $(\rho v)_w \sim \text{const.}$  and  $(\rho v)_w \sim 1/\sqrt{x}$ . The characteristic experimental temperature and concentration fields for ethyl alcohol and hexane are presented in Fig. 2. The maximum value of the temperature at the flame front reached 1313°K, when hexane was burnt, and 1473°K in the burning of ethyl alcohol. In the direction of the incoming flow the temperature gradient  $(T_y)_w$  decreased and the reaction front approached the surface of the porous plate with increasing Reynolds number  $Re_\infty$ . The change in concentrations both for ethyl alcohol and hexane is approximated by similar curves (Fig. 2).  $H_2O$  and  $CO_2$  formed according to chemical reaction (1) or (2) diffuse

from the zone of chemical reaction into the external flow and to the porous wall. Since in the 1st region the content of  $O_2$  is small, carbon dioxide ( $CO_2$ ) probably reacts with vapours of reagent  $S$  with the formation of carbon monoxide  $CO$  which diffuses to the front of the chemical reaction. On the wall the concentration of  $CO$  formed after reactions (2) and (3) is considerable (Fig. 2). The vapours of reacting liquids and oxygen ( $O_2$ ) diffuses to the region  $\eta = \eta_*$  from opposite sides. Oxygen  $O_2$  is supplied out of the external flow to the reaction front where it is almost completely spent. However asymptotic decrease of oxygen content is observed up to the wall. The concentration of component  $S$  in the region  $\eta = \eta_*$  is not equal to zero and decreases smoothly in the 2nd region. The content of other gas components changes similar to  $O_2$  and  $S$ . This is due to the fact that the rate of homogenous chemical reactions is very great but not equal to infinity as it was assumed for simplicity in the mathematical analysis. In the case considered the theoretical model of the flame front (of reaction) agrees however satisfactorily with experiment since the zone, where chemical conversions are very intense, is rather narrow. This is also confirmed by the fact that despite the absence of breaks in the first derivatives of the experimental curves of temperature and concentration, the change of  $T$  and  $\eta_i$  in the reaction zone is very sharp (Fig. 2). As it was already noted, the position of the reaction front determined in the experiment agrees well with the theory.

Heat transfer coefficients  $St$ , mass-transfer coefficients  $St_m$  in dimensionless form

$$St = \frac{q_w}{\rho_\infty u_\infty c_{p\infty} (T_* - T_w)},$$

$$St_m = \frac{m_\infty}{m_s} \cdot \frac{(\rho v)_{ws}}{\rho_\infty u_\infty} \cdot \frac{1}{\ln(1 - P_{ws}) - 1} \quad (24)$$

and the numbers  $Pr$  and  $Sc_S$  for the multi-component mixture were determined experimentally;  $q_w$ , according to (25). All thermal

parameters were calculated at the surface of the body, in the reaction zone and at external edge of the boundary layer, and then their arithmetic mean value was taken. Such averaging leads to the best correlation of experimental points. The graphical presentation of functions  $St = St(Re, Pr)$  and  $St_m = St_m(Re, Sc_i)$  in logarithmic co-ordinates showed that the scatter of experimental points was rather significant. Consequently, these dimensionless expressions must include an additional number which would account for the peculiarities of the heat and mass-transfer process with chemical reactions in the boundary layer.

As the analysis of the experimental and theoretical data showed, a modified temperature factor is such a number

$$\frac{\lambda_w}{\lambda_*} \cdot \frac{T_w}{T_*} \quad (25)$$

When concentration in the boundary layer is constant, the coefficients  $\lambda_w$  and  $\lambda_*$  are power functions of temperature  $T$  and therefore number (25) is of the form of the ordinary temperature factor  $T_w/T_\infty$ , when  $T_* = T_\infty$ . The number (25) can also be obtained from condition (13) under

the assumption that the heat evolved due to chemical reaction is supplied by heat conduction to the surface of the body [7].

Under the condition considered  $K$  was assumed to be of the form  $K = (\pi_w/\pi_*)T_\infty/T_*$ , as the value  $T_w$  in the authors experiments was constant (somewhat less than the boiling point of the injected fluid  $S$ ). Thus experimental data were correlated in the form

$$St = f(Re, Pr, K), St_m = f(Re, Sc_i, K). \quad (26)$$

The exponent of the number  $K$  was shown to be equal to  $\frac{1}{3}$ .

The results of the treatment for the case  $(\rho v)_w = \text{const.}$  are presented in Fig. 3. For injection in accordance with the relation  $(\rho v)_w \approx 1/\sqrt{x}$  graphs are of the form similar to that in Fig. 3. The empirical dimensionless equations obtained for local  $St$  and  $St_m$  are of the form

$$St = ARe^{-\frac{1}{2}}Pr^{-\frac{1}{2}}K^{\frac{1}{3}}, \\ St_m = BRe^{-\frac{1}{2}}Sc^{-\frac{1}{2}}K^{\frac{1}{3}} \quad (27)$$

where at  $(\rho v)_w \sim \text{const.}$   $A = 0.77$ ,  $B = 5.78$ ; at  $(\rho v)_w \sim 1/\sqrt{x}$ ,  $A = 0.66$ ,  $B = 4.99$ .

Introduction of the number  $K$  into the correlating equations yielded satisfactory correlation of experimental points whose scatter near the straight lines did not exceed  $\pm 18$  per cent (Fig. 3). For both methods of injection the absolute values of the numbers  $St$  (or  $St_m$ ) are practically the same,  $St_m$  however being higher than  $St$ .

The experimental points characterizing the mean heat and mass transfer with a scatter of  $\pm 15$  per cent are approximated by relationships of the form

$$\overline{St} = \overline{A}Re^{-\frac{1}{2}}Pr^{-\frac{1}{2}}K^{\frac{1}{3}}, \quad \overline{St}_m = \overline{B}Re^{-\frac{1}{2}}Sc^{-\frac{1}{2}}K^{\frac{1}{3}}. \quad (28)$$

Here  $\overline{Re} = u_\infty L/\nu_\infty$ , where  $L$  is total length of the plate. At  $(\rho v)_w \sim \text{const.}$ ,  $\overline{A} = 1.19$ ,  $\overline{B} = 8.91$ ; at  $(\rho v)_w \sim 1/\sqrt{x}$ ,  $\overline{A} = 1.08$ ,  $\overline{B} = 7.2$ . In relationships (28) both coefficients  $\overline{A}$  are almost similar but  $\overline{St}_m > \overline{St}$ . As follows from (27) and (28), the numbers  $St_m > St$  are several times larger than  $St$  and  $\overline{St}$ . This can be explained by the adopted

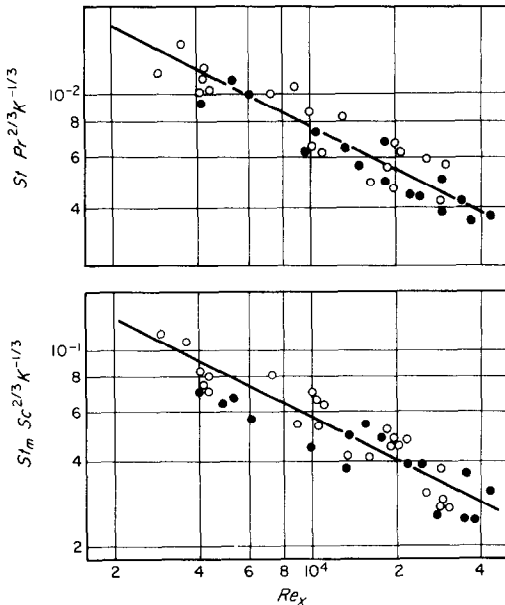


FIG. 3.  $StPr^{\frac{2}{3}}K^{-\frac{1}{3}}$  and  $St_mSc^{\frac{2}{3}}K^{-\frac{1}{3}}$  vs.  $Re_x$ .



methods of calculation of dimensionless mass-transfer coefficients according to (25). In the authors' experiments  $P_{ws}$  did not exceed 0.16, hence the absolute values of  $St_m$  appeared greater than  $St$ . However, already at  $P_{ws} = 0.55$  the numbers  $St$  and  $St_m$  [and therefore coefficients  $A$  and  $B$  in (27) and (28)] are similar.

Due to the absence of published experimental data on evaporation (without burning) or hexane and ethyl alcohol from the surface of a porous plate, equations (27) and (28) obtained for  $\overline{St}$  were transformed to the form

$$St = cRe^{-\frac{1}{2}}Pr^{-\frac{1}{3}}, \quad \overline{St} = \overline{c}Re^{-\frac{1}{2}}Pr^{-\frac{1}{3}} \quad (29)$$

where at  $(\rho v)_w \sim \text{const.}$ ,  $C = 0.31$ ,  $\overline{C} = 0.50$ ; at  $(\rho v)_w \sim 1/\sqrt{x}$ ,  $C = 0.30$ ,  $\overline{C} = 0.48$ , and compared with the relationships for a dry body

$$St_{dr} = 0.332 Re^{-\frac{1}{2}}Pr^{-\frac{1}{3}}, \quad \overline{St}_{dr} = 0.664 \overline{Re}^{-\frac{1}{2}}Pr^{-\frac{1}{3}} \quad (30)$$

where all the parameters refer to  $T_\infty$ . Comparison of equations (29) and (30) shows that the ratios  $St/St_{dr}$  and  $St/\overline{St}_{dr}$  change from 0.90 to 0.93 and from 0.72 to 0.75, respectively. Thus, for the investigated injection intensities  $(\rho v)_w$  heat transfer with chemical reaction is less than heat transfer of a "dry" body. This can perhaps be explained by introduction of the injected fluid  $S$  and the presence of the already mentioned endothermic conversions of type (3) or (4) at the

wall. Evidently, the lower value of the ratio  $\overline{St}/\overline{St}_{dr}$  in comparison with  $St/St_{dr}$  is due to the less intense heat transfer in the 1st section, i.e. to some deviation of the experimental curve  $St = St(x)$  from the theoretical curve (30) for  $St_{dr}$  where  $St_{dr}$  is a hyperbolic function of  $x$ .

As follows from the analysis of the experimental curves of  $Le_S = f(\eta)$ , presented for the second section of the experimental plate in Fig. 4, within the investigated range of temperature ( $T_\infty = 290\text{--}473^\circ\text{K}$ ) and velocity ( $R_\infty = 0.2\text{--}0.6 \cdot 10^5$ ) of flow at  $(\rho v)_w \sim \text{const.}$  and  $(\rho v)_w \sim 1/\sqrt{x}$  the Lewis numbers  $Le_S = Pr/Sc_S$  are significantly different from unity. Only at  $\eta \rightarrow 1$ ,  $Le_S \rightarrow 1$ . For hexane and ethyl alcohol  $Pr_\infty \approx Pr_w \approx 0.7$  the values of  $Pr_*$  did not exceed 0.8. Thus, when the temperature  $T$  of the boundary layer changes over a wide range (from 290 to  $1400^\circ\text{K}$ ), the changes in the value of  $Pr$  with  $T$  are less noticeable. For ethyl alcohol  $Sc_{aw}$  and  $Sc_{a*}$  are approximately 1.7 times higher than the corresponding values of  $Pr$  and  $Pr_*$ . However, for hexane  $Sc_{h*}$  is 2.2 times greater than  $Pr_*$ . Consequently, at all values of  $\eta = u/u_\infty$  (except  $\eta = 1$ ) the values  $Le_S$  differ from unity. Besides, the assumption of constant values of the numbers  $Pr$  and  $Sc_S$  over the thickness of the reacting boundary layer made by some authors [1] may be valid in some particular cases.

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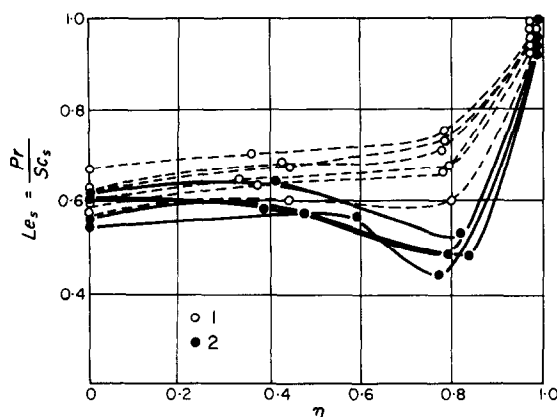


FIG. 4. Distribution of  $Le_S$  numbers in the boundary layer.

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#### HEAT AND MASS TRANSFER FOR REACTION OF INJECTED FLUID WITH EXTERNAL OXYGEN FLOW

**Abstract**—Heat and mass transfer in a laminar boundary layer with injected liquid (hexane and ethyl alcohol) reacting with the oxygen of the external flow after evaporation is studied theoretically and experimentally. The system of differential equations of transfer in the boundary layer is solved for the case of an infinitely fast rate of homogeneous reaction including the effect of multicomponent diffusion and of variable and unequal numbers of  $Pr$  and  $Sc_2$ . Temperature, velocity and concentration profiles in the boundary layer of a four-sectional porous plate were determined experimentally and the intensity of local and mean heat and mass transfer rates along the body evaluated.

#### TRANSFERT DE CHALEUR ET DE MASSE DANS UNE RÉACTION ENTRE UN LIQUIDE INJECTÉ ET L'OXYGÈNE D'UN ÉCOULEMENT GAZEUX

**Résumé**—Le transport de chaleur et de masse dans une couche limite laminaire avec un liquide injecté (hexane et alcool éthylique) réagissant après évaporation avec l'oxygène de l'écoulement extérieur est étudié théoriquement et expérimentalement. Le système d'équations différentielles de transport dans la couche limite est résolu dans le cas d'une vitesse de réaction homogène infiniment grande par rapport à la diffusion de plusieurs constituants pour des nombres  $Pr$  et  $Sc$  variables et inégaux. Les champs de température, de vitesse et de concentration dans la couche limite d'une plaque poreuse à quatre sections sont déterminés expérimentalement et l'on trouve alors l'intensité du transport de chaleur et de masse, local et moyen, le long du corps.

#### WÄRME- UND STOFFÜBERGANG BEI DER REAKTION ZWISCHEN EINEM INJEZIERTEN FLUID UND EINEM SAUERSTOFFSTROM

**Zusammenfassung**—Es wird theoretisch und experimentell der Wärme- und Stoffübergang in einer laminaren Grenzschicht untersucht, in die Flüssigkeit (Hexan und Äthylalkohol) eingespritzt werden und nach der Verdampfung mit dem vorbeiströmenden Sauerstoff verbrennen. Das System der Differentialgleichungen für die Transportvorgänge in der Grenzschicht wird gelöst für den Fall der unendlich raschen homogenen Reaktion; dabei wird die Diffusion mehrerer Komponenten für variable und nicht übereinstimmende Zahlenwerte von  $Pr$  und  $Sc$  berücksichtigt. Es werden das Temperatur-, das Geschwindigkeits- und das Konzentrationsfeld in der Grenzschicht einer porösen Platte aus vier Abschnitten experimentell bestimmt. Damit lässt sich die Grösse der lokalen und der mittleren Wärme-, bzw. Stoffübergangszahl finden.